

The University of Chicago

THE INVARIANCE OF FACTORIAL COMPOSITION OF A TEST

A DISSERTATION SUBMITTED TO THE FACULTY
OF THE DIVISION OF THE BIOLOGICAL SCIENCES
IN CANDIDACY FOR THE DEGREE OF DOCTOR
OF PHILOSOPHY

DEPARTMENT OF PSYCHOLOGY

AUGUST, 1940

By
LORENZ A. MEYER

CHICAGO, ILLINOIS

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ACKNOWLEDGMENT

The writer is indebted to Professor L. L. Thurstone for suggesting the topic of this study and for his generous aid and supervision during the course of the investigation, and to Mr. Ledyard Tucker for his willing and helpful assistance.



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I. INTRODUCTION

The invariance of factorial composition of a test has been proposed by Professor L. L. Thurstone (7) as a fundamental criterion for an acceptable factor method. He writes: "One of the fundamental requirements of a successful factorial method is that the factorial description of a trait must remain invariant when the trait is moved from one battery to another which involves the same common factors or abilities."

Thurstone has in other writings (9) and in his lectures expanded this statement. He believes that, if an underlying order is present in a set of tests, a proper factorial method must discover that order, it must arrive at the loadings of the tests on the underlying factors; that such a method will also determine the same factorial composition for a test when it is moved from one battery to another which contains the same common factors. Furthermore, if a method fails to discover the underlying factors or destroys the invariance of the loadings, it must be considered questionable as a factor method. If it alters the loadings significantly it is to that extent inadequate.

Thurstone's view is also that a factor method when used with psychological data should give psychologically meaningful results, and that he has developed a method which yields such results; more specifically, if there are underlying factors or functional unities, they can be isolated by his method provided certain conditions are met, which are designated by the term "Simple Structure." The theory is further that these conditions are inherent in certain groups of tests; that there is in the nature of tasks used as tests or included in tests, a condition or series of conditions which make possible the determination of underlying and invariant factors.

These conditions are that for a given functional unity there are some activities, tasks, or tests, the performance of which does not involve that ability. Then, geometrically, each vector representing a test lies in at least one of the bounding hyperplanes, and for a large enough battery there will be at least

one vector to define a hyperplane in each of the necessary $r-1$ dimensions. For a given column there will be a singular zero¹ relative to each other column. (The more tests that are used for a given region--that is, a region in which the number of functional unities remains constant--the more likely one is, of course, to have such "plane" tests, especially if no attempt is made to make the tests complex.)

These conditions make it possible to determine the bounding hyperplanes of the positive manifold, which, it is reasonable to assume, exists for psychological tests. If the conditions of Simple Structure do not exist or if the structure is not clear, no unique solution is possible. Whether a Simple Structure exists in any given test battery is a question of fact.

The determination of the hyperplanes can be made by Thurstone's Rotational Procedure after extraction of the Centroid Axes.

II. OTHER STUDIES

A few other studies of the problem of finding invariance of factorial composition have been made. Smart (3) found that first factor loadings determined for a given variable by means of Thurstone's Centroid Method change considerably when included in different batteries containing different variables. The amount of variation was even greater for second factor loadings.

Humphreys (1) made a Centroid Analysis of a group of 21 variables, of a second group containing 14 of these variables, and of a third group containing 15 of these variables. He found that the factorial compositions were stable. However, one would not, in general, expect to find such stability with factors determined by the Centroid Method.

Sisk (2) determined the rotated factorial matrix for the five parts of the American Council Examination, for nine engineering courses, and for the 14 variables combined. The rotated values "were computed by means of the method described by Peters." He found "there is a variation in the factorial description of each measure when the number of measures in the correlational matrix is varied." His rotated factorial matrices (Table 6, p. 172)

¹A singular zero for column A relative to column B would be a zero entry in column A in any row in which the entry in column B is non-vanishing.

show that either his batteries did not satisfy the conditions of Simple Structure, or he did not rotate to Simple Structure. Hence invariance would not be expected for his batteries.

Thomson (4) has questioned the existence of functional unities. He writes, "My own belief is that the mind is not divided up into 'unitary factors,' but is a rich, comparatively undifferentiated complex of innumerable influences." In another article (5) he specifically questions the likelihood of finding invariance, "I do not think that it [the factorial composition] will naturally remain invariant. If however such invariance is found to occur naturally, without undue forcing of the sets of tests, then this will be a very important observation. So far the evidence of this appears to me to be inadequate or adverse." In his recent book (6) he expresses doubt about the possibility of there being unique factors, but does not rule out such a possibility. He writes, "if the resulting 'simple structure' factors [from different batteries] were compatible with one another and in fair agreement with psychological intuition, there would be formed an apparatus for defining factors which would have considerable influence on the progress of psychology."

III. PROBLEM

The purpose of this study was to determine whether by means of Thurstone's Factor Method the invariance of factorial composition of a test is preserved when it is moved from one battery to another; in other words, whether Thurstone's Method satisfies the criterion of invariance which he has set up.

IV. PROCEDURE

Plan of the study.--The plan of the study was to select three batteries of about 20 tests each from a large battery of tests which had been given to the same population and which satisfied the conditions of Simple Structure. All batteries were to be factored and rotated by Thurstone's Method. The comparison of the factorial compositions of each test which occurred in the large battery and in one or more of the smaller batteries would show whether invariance was preserved.

Data.--The data used were a correlational matrix derived from a set of 61 tests given to Eighth Grade children in a study

made by Professor L. L. Thurstone. The matrix had already been factored and rotated in that study previous to the selection of the batteries for this study.

Selection of the batteries.---The batteries were selected according to the following conditions:

1. The number of tests in each battery should be about 20. This limit was set because of the convenience of factoring and handling other calculations for that number of tests. The actual number included in each battery was for Batteries I, II, and III respectively 20, 21, and 19.

2. If a factor was to be present in a battery there should be at least three tests in which that factor had loadings of .40 or more.

3. The batteries should differ as much as possible. The differences were achieved by varying in the several batteries the number of tests which had loadings on a given factor. In each battery tests were included so that a large number (8 or 10) had loadings on one factor.

4. The tests were selected so that some factors and some tests would be common to two or all three of the small batteries, in order to make possible the comparison of corresponding factor loadings in these batteries. It was, of course, not possible in a battery of 20 tests to have all 10 factors adequately represented.

The tests selected for each of the three batteries and their loadings on the factors in the original Total Battery are listed in Tables 1-I, 1-II, and 1-III. The original numbers and names of the tests are used.

Tables 2-I, 2-II, and 2-III are the Correlational Matrices for the three batteries. The numbers of the tests are the same as those used by Professor Thurstone in the original study.

Centroid analysis.---The tests were coded, that is, new numbers were assigned to the tests in each battery; and these were used instead of the names or original numbers of the tests in order to avoid the operation of any possible tendency to determine the factors by means of knowledge of test content.

Seven factors were extracted from each battery by Thurstone's Centroid Method (7). In changing signs the new method making use of column sums developed by Mr. Ledyard Tucker was used. After extraction of the seventh factor the largest residual was coincidentally .053 for each of the three batteries. Tables 3-I,

3-II, and 3-III are the Centroid Factorial Matrices obtained.

TABLE 1-I

TESTS SELECTED FOR BATTERY I AND THEIR LOADINGS
FROM ROTATED FACTORIAL MATRIX (V)
FOR TOTAL BATTERY

Test Number		Factors									
		N	W	S	V	M	I	P	X ₁	X ₂	X ₃
N Tests											
3..	Addition	.44	...	...	...	...	...	...	.32	..	...
37..	Multiplication	.46	...	...	...	...	...	.24	.26	..	...
56..	Three Higher	.41	...	...	...	...	...	.21	...	..	...
W Tests											
4..	Anagrams	...	.41	...	...	...	...	.23	...	..	...
6..	Association	...	.39	...	...	...	...	...	...	..	.28
24..	First Letter	...	.59	...	...	...	...	...	...	..	.20
42..	Prefixes	...	.61	...	...	...	...	...	...	..	...
50..	Rhyming	...	.52	...	...	...	...	...	...	..	.21
54..	Suffixes	...	.58	...	...	...	...	...	...	..	...
55..	Synonyms	...	.51	...	...	...	...	...	...	..	.20
60..	Word Puzzles	...	.39	...	...	...	...	.37	...	..	...
S Tests											
8..	Cards	...	...	.68	...	...	...	...	...	..	...
22..	Figures	...	...	.62	...	...	...	...	...	..	...
26..	Flags	...	...	.52	...	...	...	...	...	..	...
P Tests											
7..	Mirror Reading	...	.20	...	...	...	...	.40	...	..	...
18..	Faces	...	...	.27	...	...	...	.32	...	..	...
30..	Identical Numbers	.22	...	...	...	...	...	.45	...	..	...
X ₁ Tests											
14..	Dot Counting I	...	...	...	...	...	...	...	.69	..	...
15..	Dot Counting II	...	...	...	...	...	...	...	.54	..	...
16..	Dot Counting III	...	...	...	...	...	...	...	.70	..	...

TABLE 1-II

TESTS SELECTED FOR BATTERY II AND THEIR LOADINGS
FROM ROTATED FACTORIAL MATRIX (V)
FOR TOTAL BATTERY

Test Number		Factors									
		N	W	S	V	H	I	P	X ₁	X ₂	X ₃
N Tests											
5..	Arithmetic	.36	...	...	.20	...	.22	...	...	...	...
37..	Multiplication	.46	...	...	...	...	...	.24	.26	...	...
56..	Three Higher	.41	...	...	...	...	...	.21	...	...	...
W Tests											
23..	First and Last Letter	...	.58	...	...	...	...	...	...	...	...
27..	Four-letter Words	...	.53	...	...	...	...	...	...	...	...
42..	Prefixes	...	.61	...	...	...	...	...	...	...	...
V Tests											
2..	Absurdities	...	...	...	.35	...	...	...	...	...	...
39..	Paragraph Recall	...	...	...	.55	...	...	...	...	...	...
43..	Proverbs	...	...	...	.40	...	...	...	...	...	...
45..	Reading Vocabulary	...	...	...	.68	...	...	...	...	...	.22
46..	Reading Sentences	...	...	...	.64	...	...	...	...	...	.27
47..	Reading Paragraphs	...	...	...	.53	...	...	...	...	...	.31
48..	Reading Paragraphs	...	...	...	.45	...	...	...	...	...	.24
51..	Same-Opposite	...	...	...	.62	...	...	...	...	...	.25
S Tests											
8..	Cards	...	...	.68	...	...	...	...	...	...	...
22..	Figures	...	...	.62	...	...	...	...	...	...	...
26..	Flags	...	...	.52	...	...	...	...	...	...	...
X ₂ Tests											
28..	Geometrical Forms	...	...	.22	...	...	.21	...	...	.23	...
35..	Mazes I	...	...	.28	...	...	...	...	...	.60	...
36..	Mazes II	...	...	.33	...	...	...	...	...	.54	...
44..	Pursuit	...	...	.20	...	...	...	...	.25	.29	...

TABLE 1-III
TESTS SELECTED FOR BATTERY III AND THEIR LOADINGS
FROM ROTATED FACTORIAL MATRIX (V)
FOR TOTAL BATTERY

Test Number		Factors									
		N	W	S	V	M	I	P	X ₁	X ₂	X ₃
V Tests											
10..	Completion	...	...	...	.55	...		(.17)	...		.20
45..	Reading Vo- cabulary	...	...	...	.68	...			...		.22
51..	Same-Opposite	...	...	...	.62	...			...		.25
S Tests											
8..	Cards	...	...	.68	...	...			...		...
22..	Figures	...	...	.62	...	...			...		...
26..	Flags	...	...	.52	...	...			...		...
P Tests											
7..	Mirror Reading	...	.20	...	...	...		.40	...		...
13..	Dis. Sen- tences	...	...	...	.28	...		.27	...		.27
17..	Dot Patterns	.27	...	...	...	...		(.19)	.31		...
19..	Figure Group- ing	...	...	...	...	...	(.18)	.25	...		...
20..	Figure Naming	...	...	...	...	.29		.23	...	(.18)	.28
30..	Identical Numbers	.22	...	...	...	...		.45	...		...
31..	Identical Pictures	...	...	...	...	...		.36	...		...
32..	Incomplete Words	...	.29	...	...	...		.33	...		.30
38..	Number Pat- terns	...	...	...	...	...	.26	.24	...		...
52..	Scattered X's	...	...	...	...	...		.27	...		...
X ₃ Tests											
41..	Picture Nam- ing	...	...	...	...	...		(.19)	...	.27	.35
57..	Verbal Enu- meration	...	...	...	.38	...		.42	...		.48
58..	Word Checking	...	...	...	.32	...		(.19)	...		.36

TABLE 2-1
CORRELATIONAL MATRIX FOR BATTERY I

	3	4	6	7	8	14	15	16	18	22	24	26	30	37	42	50	54	55	56
2	267																		
3	257	399																	
4	232	418	307																
5	126	132	075	276															
6	383	158	202	227	054														
7	271	151	186	251	066	503													
8	332	122	184	489	376	699	598												
9	152	302	256	213	027	272	353	313											
10	097	133	062	213	106	027	102	038	319										
11	248	494	536	385	106	210	179	211	264	120									
12	085	147	127	271	623	088	171	096	337	510	143								
13	284	315	219	431	114	355	388	318	393	044	264	086							
14	499	317	321	307	023	416	296	333	235	007	310	083	467						
15	165	374	418	289	032	101	095	124	141	078	506	133	192	177					
16	193	423	484	404	102	165	108	146	209	102	563	178	216	237	447				
17	225	367	396	337	059	155	115	134	187	080	530	122	216	262	447	487			
18	125	275	451	184	012	075	051	104	125	041	490	090	216	262	554	462	395		
19	394	301	279	364	307	276	268	210	309	292	256	328	291	436	445	238	221	137	
20	279	533	461	526	107	233	201	224	303	122	565	174	369	353	387	516	505	305	379

TABLE 2-II
CORRELATIONAL MATRIX FOR BATTERY II

	2	5	8	22	23	26	27	28	35	36	37	39	42	43	44	45	46	47	48	51
2																				
5	.284																			
8	.185	.269																		
22	.228	.257	.621																	
23	.170	.205	.110	.109																
26	.265	.313	.623	.510	.147															
27	.170	.254	.115	.172	.514	.188														
28	.238	.248	.384	.322	.107	.354	.065													
35	.148	.102	.286	.206	.160	.261	.054	.318												
36	.129	.153	.344	.256	.111	.329	.066	.324	.684											
37	.077	.208	.023	.007	.239	.083	.221	.052	.039	.042										
39	.388	.444	.156	.170	.178	.218	.261	.168	.077	.066	.183									
42	.120	.148	.032	.078	.461	.133	.408	.035	.096	.057	.177	.180								
43	.340	.389	.197	.253	.244	.280	.336	.226	.101	.099	.063	.431	.172							
44	.095	.143	.315	.218	.125	.322	.086	.346	.437	.386	.122	.109	.086	.078						
45	.420	.430	.158	.211	.237	.231	.304	.169	.043	.064	.133	.624	.227	.551	.063					
46	.394	.420	.189	.268	.212	.242	.291	.177	.020	.061	.172	.623	.189	.492	.071	.769				
47	.342	.360	.144	.207	.206	.159	.227	.104	.039	.001	.157	.547	.184	.439	.015	.671	.681			
48	.334	.350	.107	.212	.116	.148	.170	.101	.028	.018	.157	.495	.115	.364	.001	.553	.567	.512		
51	.397	.365	.151	.178	.245	.195	.306	.171	.059	.040	.210	.550	.241	.476	.106	.730	.661	.579	.520	
56	.212	.454	.307	.292	.200	.328	.230	.261	.170	.200	.436	.324	.176	.269	.181	.281	.331	.258	.240	.310

TABLE 2-III
CORRELATIONAL MATRIX FOR BATTERY III

	7	8	10	13	17	19	20	22	26	30	31	32	38	41	45	51	52	57
7																		
8	.276																	
10	.340	.206																
13	.378	.149	.546															
17	.372	.242	.138	.191														
19	.314	.295	.148	.192	.246													
20	.275	.072	.192	.280	.271	.127												
22	.213	.621	.223	.140	.187	.168	.016											
26	.271	.623	.272	.205	.282	.227	.006	.510										
30	.431	.114	.184	.237	.454	.204	.386	.044	.086									
31	.379	.296	.186	.229	.332	.324	.225	.209										
32	.488	.092	.337	.404	.313	.171	.288	.078	.116	.397	.185	.192						
38	.294	.186	.202	.254	.277	.223	.180	.102	.211	.368	.286	.369	.232					
41	.218	.030	.236	.333	.294	.224	.415	.040	.056	.361	.243	.257	.048	.066				
45	.209	.158	.718	.454	.042	.038	.075	.211	.231	—	.078	.292	.112	.147	.730			
51	.240	.151	.672	.497	.078	.099	.148	.178	.195	.126	.140	.132	.100	.074	—	-.063		
52	.234	.123	—	.006	.141	.124	.100	.063	.094	.304	.192	.494	.224	.432	-.161	.500	.072	
57	.385	.142	.547	.541	.284	.168	.321	.094	.144	.407	.350	.347	.473	.456	.439	.473		
58	.301	.219	.485	.440	.282	.164	.219	.236	.227	.173	.227		.152	.273	.456		-.011	.557

TABLE 3-I
CENTROID FACTORIAL MATRIX FOR BATTERY I

	I	II	III	IV	V	VI	VII
3.....	.486	-.156	-.247	.174	-.285	.187	.043
4.....	.586	.235	.109	.229	.081	.067	-.060
6.....	.585	.336	-.066	.034	-.068	.045	-.188
7.....	.634	-.030	.200	.207	.247	-.125	.136
8.....	.380	-.380	.533	-.257	-.147	-.030	-.078
14.....	.503	-.344	-.482	-.098	.149	.213	.104
15.....	.481	-.379	-.315	-.099	.193	-.088	-.081
16.....	.494	-.345	-.475	-.208	.265	.129	.026
18.....	.553	-.253	.175	.045	.208	-.193	-.157
22.....	.345	-.284	.506	-.259	-.161	.055	-.107
24.....	.660	.423	-.024	-.037	.040	.062	-.092
26.....	.420	-.278	.469	-.285	-.152	-.054	.054
30.....	.525	-.176	-.152	.285	.157	-.213	.039
37.....	.542	-.136	-.301	.361	-.212	.074	.123
42.....	.512	.445	-.060	-.154	-.098	-.198	.102
50.....	.593	.410	.060	-.043	.055	.053	.056
54.....	.561	.413	-.039	-.076	-.053	-.104	.216
55.....	.426	.444	-.067	-.180	-.086	-.026	-.115
56.....	.561	-.201	.096	.187	-.247	.092	.062
60.....	.675	.253	.082	.211	.156	.133	.149

TABLE 3-II
CENTROID FACTORIAL MATRIX FOR BATTERY II

	I	II	III	IV	V	VI	VII
2.....	.501	.082	.150	-.085	.075	-.096	-.093
5.....	.585	.056	.062	.119	-.186	-.242	-.023
8.....	.499	-.522	.239	.276	.062	.158	.029
22.....	.505	-.377	.248	.269	.075	.113	.134
23.....	.431	.149	-.461	.185	.258	.041	-.073
26.....	.557	-.417	.156	.261	.063	.056	-.094
27.....	.463	.218	-.357	.270	.239	-.023	.019
28.....	.416	-.364	.116	-.073	.040	-.120	-.187
35.....	.363	-.534	-.290	-.409	.097	-.078	.094
36.....	.374	-.572	-.194	-.348	.096	-.130	.182
37.....	.285	.116	-.342	.153	-.456	.138	.075
39.....	.638	.307	.161	-.131	-.148	-.095	-.085
42.....	.353	.176	-.406	.133	.196	.127	-.122
43.....	.594	.188	.143	.033	.152	-.202	.057
44.....	.350	-.424	-.153	-.170	-.062	.099	-.179
45.....	.714	.421	.239	-.171	.074	.059	.076
46.....	.711	.373	.249	-.128	-.064	.093	.090
47.....	.602	.400	.226	-.089	-.059	.116	.068
48.....	.522	.336	.252	-.076	-.120	.030	.036
51.....	.675	.376	.115	-.152	.015	.101	.053
56.....	.553	-.079	-.125	.210	-.379	-.132	.116

TABLE 3-III
CENTROID FACTORIAL MATRIX FOR BATTERY III

	I	II	III	IV	V	VI	VII
7.....	.645	.137	.198	-.213	-.039	.194	.093
8.....	.480	.455	-.446	.162	.101	.134	.060
10.....	.654	-.434	-.270	-.120	-.143	-.031	-.123
13.....	.625	-.317	.019	.057	-.137	.100	.033
17.....	.508	.255	.192	.044	.129	-.110	-.180
19.....	.393	.243	.061	.065	-.207	.078	.090
20.....	.417	-.081	.332	.104	.114	.047	-.145
22.....	.403	.330	-.497	.068	.147	.088	.064
26.....	.483	.369	-.432	.081	.014	.047	-.078
30.....	.522	.170	.454	-.140	.118	-.215	-.096
31.....	.514	.283	.140	.061	-.172	-.197	.108
32.....	.563	-.154	.252	-.137	.163	.183	.078
38.....	.403	.169	.151	.038	-.194	-.024	-.112
41.....	.467	-.123	.409	.245	.050	.111	-.116
45.....	.496	-.510	-.445	-.190	-.120	-.059	-.044
51.....	.566	-.489	-.307	-.176	-.102	-.107	.021
52.....	.182	.310	.201	-.239	.126	-.037	.141
57.....	.692	-.340	.150	.061	.088	-.183	.177
58.....	.601	-.270	-.110	.161	.090	-.120	.155

Rotational procedure.--By means of Thurstone's Rotational Procedure (8), in each battery seven planes were located so as to conform to the configuration of points representing the test vectors. To determine the projections on the reference vectors the Centroid Matrix is multiplied by the transformation matrix. This is expressed by the matrix equation,

$$V = F_c M$$

The number of rotations necessary for each battery were: Battery I--9, Battery II--10, and Battery III--14. Tables 4-I, 4-II, and 4-III are the final transformation matrices.

V. RESULTS

Table 5 is the table of factor loadings for all batteries, that is, for the Total Battery and for Batteries I, II, and III. The table is a combination of the Rotated Factorial Matrices V_T , V_I , V_{II} , and V_{III} . The factors which were not identified in Battery I are a_1 and a_2 . The factor not identified in Battery II is b and the factor not identified in Battery III is c .

Table 6 is the table of cosines (Λ/Λ) between the reference vectors for all batteries. The deviations of the cosines in the small batteries from those in the large battery are, in general, not extreme. The largest change is in the cosine between N and X_1 , which in the Total Battery is -.05 and in Battery I is -.43.

Table 7 is the table of cosines, $R_p = D(\Lambda/\Lambda)^{-1}D'$, between the primary vectors for all batteries. It is a combination of the matrices R_{pT} , R_{pI} , R_{pII} , and R_{pIII} . In this table one of the two largest changes is again in the cosine between N and X_1 (Total Battery .06, Battery I .54). The other is between N and P (Total Battery -.06, Battery I .43).

TABLE 4-I
TRANSFORMATION MATRIX Λ_9 FOR BATTERY I

	A	B	C	D	E	F	G
I.....	.186	.369	.195	.267	.157	.134	.117
II.....	.076	.730	-.366	-.444	-.210	-.138	.116
III.....	.399	-.110	-.484	.696	-.163	-.086	.062
IV.....	.567	-.440	-.464	-.369	.566	.141	-.191
V.....	.679	-.159	.522	-.332	-.744	.100	.098
VI.....	.137	-.093	.324	.018	.093	-.946	-.202
VII.....		-.304		.014	.147	-.174	.939

TABLE 4-II
TRANSFORMATION MATRIX Λ_{10} FOR BATTERY II

	A	B	C	D	E	F	G
I.....	.404	.188	.175	.161	.099	.137	.082
II.....	.460	-.443	.190	-.386	-.001	.014	-.087
III.....	.422	.446	-.606	-.332	-.328	.014	.042
IV.....	-.528	.618	.332	-.700	.263	.216	-.202
V.....	.017	.141	.575	.110	-.884	-.131	-.230
VI.....	.350	.394	.172	.046	-.015	-.944	.177
VII.....	.212	.117	-.309	.459	.180	-.161	-.927

TABLE 4-III
TRANSFORMATION MATRIX Λ_{14} FOR BATTERY III

	A	B	C	D	E	F	G
I.....	.409	.117	.194	.142	.217	.183	.150
II.....	-.654	.280	.405	.172	-.228	-.106	.021
III.....	-.333	.163	-.553	.201	.185	.277	.175
IV.....	-.477	-.076	.292	-.820	.645	.046	.051
V.....	-.160	-.886	.574	.442	.550	-.009	.079
VI.....	-.195	-.084	.183	.135	-.337	.936	-.164
VII.....	.054	.290	-.208	.154	.188	.023	-.955

TABLE 5

FACTOR LOADINGS OF EACH TEST IN ALL BATTERIES
IN WHICH IT WAS INCLUDED*

(Combination of Matrices V_T , V_I , V_{II} , and V_{III})

Test	Bat- tery	N	W	S	V	M	I	P	X_1	X_2	X_3	a_1	a_2	b	c
1..	T	16	-05	04	15	17	12	04	25	03	-04	...	...	...	...
2..	T	-12	-01	04	35	05	16	03	-03	07	11	...	...	...	...
	II	-10	04	03	30	...	15	...	...	02	...	...	...	11	...
3..	T	44	04	02	00	00	-04	03	32	-13	-02	...	...	...	...
	I	48	03	06	...	...	...	-09	10	...	...	-08	-04	...	...
4..	T	11	41	-01	-02	06	09	23	-07	-05	00	...	...	...	...
	I	09	27	02	...	...	...	36	-07	...	...	02	00	...	...
5..	T	36	-01	08	20	06	22	-09	-04	08	-08	...	...	...	...
	II	23	-03	06	13	...	36	...	...	-07	...	...	...	04	...
6..	T	03	39	-04	00	04	15	04	05	05	28	...	...	...	...
	I	08	52	-03	...	...	...	09	-01	...	...	03	-10	...	...
7..	T	00	20	12	06	09	07	40	-01	01	04	...	...	...	...
	I	02	03	16	...	...	...	46	03	...	...	22	22	...	...
	III	00	33	00	18	...	21	...	...	...	-06	...	...	...	35
8..	T	01	-06	68	-04	-01	00	05	-04	-02	01	...	...	...	...
	I	00	-03	78	...	...	...	01	-01	...	...	05	00	...	...
	II	-01	-01	68	-02	...	-04	...	...	04	...	...	...	03	...
	III	06	05	64	-07	...	02	...	...	...	04	...	...	...	00
9..	T	27	-10	20	09	26	09	06	-01	13	00	...	...	...	...
10..	T	09	06	-06	55	01	09	17	-05	12	20	...	...	...	...
	III	15	06	00	72	...	01	...	...	...	02	...	...	...	-02
11..	T	04	15	-03	06	19	12	-10	10	09	00	...	...	...	...
12..	T	16	05	-08	29	-02	30	08	04	11	16	...	...	...	...
13..	T	-02	11	-03	28	-03	14	27	00	-01	27	...	...	...	...
	III	04	25	-07	43	...	11	...	...	...	15	...	...	...	-05
14..	T	14	-01	-04	01	-05	-02	-04	69	-11	-04	...	...	...	...
	I	10	-04	-06	...	...	...	-05	65	...	...	-06	08	...	...
15	T	04	-04	06	-06	03	11	00	54	10	00	...	...	...	...
	I	-01	-02	05	...	...	...	00	50	...	...	25	-03	...	...
16	T	-04	01	04	-04	-01	-01	-06	70	-06	08	...	...	...	...
	I	-07	01	-05	...	...	...	-04	73	...	...	03	05	...	...

*All decimal points are omitted.

TABLE 5--Continued

Test	Battery	N	W	S	V	M	I	P	X ₁	X ₂	X ₃	a ₁	a ₂	b	c
17..	T III	27 32	-03 01	16 20	-06 -05	04 ...	06 00	19 ...	31 ...	03 ...	-08 19	...	...	...	...
18..	T I	-09 -06	01 01	27 29	08 ...	04 ...	07 ...	32 29	15 14	01 ...	02 ...	...	...	...	...
19..	T III	-02 -04	-04 14	15 04	-03 -03	-09 ...	18 32	25 ...	00 ...	16 ...	01 -04	...	...	...	...
20..	T III	14 26	-03 22	02 00	03 03	29 ...	-08 -08	23 ...	12 ...	18 ...	28 26	...	...	...	...
21..	T	-06	02	12	14	40	-03	-08	06	03	02	...	...	...	...
22..	T I II III	-03 -01 00 -08	00 03 -02 -01	62 72 61 59	05 ...	02 ...	-05 ...	-06 00	00 -02	-09 ...	01 ...	...	...	...	...
23..	T II	-02 00	58 62	-05 -03	-01 -05	-07 ...	07 03	07 ...	00 ...	-01 03	-10 ...	...	...	...	...
24..	T I	-01 -04	59 59	07 -03	-08 ...	-01 ...	-01 ...	07 16	01 04	-04 ...	20 ...	...	...	...	...
25..	T	03	08	-07	09	40	07	12	01	-14	18	...	...	...	...
26..	T I II III	00 00 00 08	03 04 09 -03	52 72 54 54	02 ...	-03 ...	09 ...	-04 -03	02 -01	01 ...	-04 ...	...	...	...	...
27..	T II	01 03	53 56	-02 03	05 -01	-05 ...	06 11	07 ...	-03 ...	-10 -05	-08 ...	...	...	...	...
28..	T II	-06 -08	-06 -03	22 18	12 01	08 ...	21 18	06 ...	-05 ...	23 13	-09 ...	...	...	...	...
29..	T	23	-06	25	01	03	19	14	05	11	-03	...	...	...	...
30..	T I III	22 18 29	05 -06 -01	-02 -05 -07	01 ...	09 ...	-10 ...	45 26	16 12	12 ...	01 ...	...	...	...	...
31..	T III	-10 03	-08 -08	15 00	10 02	07 ...	12 36	36 ...	03 ...	13 ...	-02 10	...	...	...	...
32..	T III	03 03	29 35	03 -02	00 26	-07 ...	05 -06	33 ...	04 ...	-11 ...	30 16	...	...	...	...
33..	T	07	13	-02	-04	06	37	15	07	-08	-01	...	...	...	...

TABLE 5--Continued

Test	Bat- tery	N	W	S	V	M	I	P	X ₁	X ₂	X ₃	a ₁	a ₂	b	c
34..	T	02	04	00	06	13	49	06	05	-12	-05	...	...	...	...
35..	T	-04	04	28	01	05	00	01	04	60	07	...	...	...	...
	II	-04	02	-08	-01	...	00	...	...	70	...	...	...	02	...
36..	T	06	02	33	-02	-02	02	02	-07	54	00	...	...	...	...
	II	-04	-07	01	-02	...	05	...	...	68	...	...	...	-07	...
37..	T	46	02	-09	02	02	-04	24	26	-08	05	...	...	...	...
	I	55	-04	-06	...	...	...	04	05	...	...	06	04	...	...
	II	60	07	-06	00	...	-01	...	...	00	...	...	...	03	...
38..	T	08	02	01	-07	05	26	24	04	05	-05	...	...	...	...
	III	19	08	-02	02	...	26	...	...	...	-02	...	...	...	-02
39..	T	11	-07	-02	55	06	02	05	02	08	18	...	...	...	...
	II	09	-05	-09	48	...	19	...	...	-04	...	...	...	16	...
40..	T	-06	03	03	17	27	41	04	-09	01	10	...	...	...	...
41..	T	-04	14	-06	00	10	01	19	16	27	35	...	...	...	...
	III	25	32	-04	-02	...	-02	...	...	...	33	...	...	...	-05
42..	T	-01	61	-01	-06	-01	-02	-06	01	07	08	...	...	...	...
	I	-02	59	-02	...	...	...	-08	-08	...	...	15	26	...	...
	II	01	56	-05	00	...	-05	...	...	00	...	...	...	06	...
43..	T	-02	10	00	40	10	13	-10	-01	00	-07	...	...	...	...
	II	-10	10	06	31	...	26	...	...	-01	...	...	...	-09	...
44..	T	-10	-04	20	13	03	01	07	25	29	-05	...	...	...	...
	II	06	05	09	-03	...	-05	...	...	31	...	...	...	29	...
45..	T	-06	02	-01	68	-02	-02	00	02	-02	22	...	...	...	...
	II	-11	03	-01	71	...	-01	...	...	04	...	...	...	-01	...
	III	02	-04	01	80	...	-04	...	...	...	-04	...	...	...	-02
46..	T	-04	-07	05	64	-07	00	04	06	-09	27	...	...	...	...
	II	03	-05	04	68	...	-02	...	...	02	...	...	...	01	...
47..	T	-02	-03	07	53	02	-02	02	-01	-11	31	...	...	...	...
	II	02	-02	03	62	...	-04	...	...	-04	...	...	...	01	...
48..	T	02	-08	07	45	01	01	-02	04	-10	24	...	...	...	...
	II	06	-10	01	53	...	05	...	...	-07	...	...	...	04	...
49..	T	26	-08	13	16	18	09	-01	-10	07	04	...	...	...	...
50..	T	-10	52	-04	11	05	06	04	00	06	21	...	...	...	...
	I	-06	50	02	...	...	...	19	00	...	...	-04	18	...	...

TABLE 5--Continued

Test	Battery	N	W	S	V	M	I	P	X ₁	X ₂	X ₃	a ₁	a ₂	b	c
51..	T	-06	03	-05	62	-01	00	12	02	00	25	...	...	...	...
	II	-02	08	-03	62	...	-04	...	...	06	...	...	...	02	...
	III	00	-04	-05	78	...	00	...	...	...	05	...	...	...	02
52..	T	03	09	10	-11	01	-09	27	-05	11	-07	...	...	...	...
	III	-06	01	02	-09	...	09	...	...	...	-04	...	...	...	39
53..	T	09	-06	20	-04	25	37	12	-09	07	08	...	...	...	...
54..	T	-04	58	00	02	05	-06	03	01	-06	09	...	...	...	...
	I	03	50	-01	...	...	...	03	-05	...	...	07	34	...	...
55..	T	-08	51	-04	-01	-04	04	-09	00	19	20	...	...	...	...
	I	-07	62	-04	...	...	...	-08	-02	...	...	01	02	...	...
56..	T	41	-05	18	05	-08	16	21	07	-03	01	...	...	...	...
	I	42	-02	32	...	...	...	08	-05	...	...	00	03	...	...
	II	51	-05	12	-01	...	27	...	...	02	...	...	...	-04	...
57..	T	-01	-03	02	38	00	-01	42	-01	13	48	...	...	...	...
	III	-01	04	-09	46	...	-01	...	...	...	44	...	...	...	06
58..	T	03	-03	15	32	02	04	19	02	03	36	...	...	...	...
	III	-05	01	11	40	...	-06	...	...	...	40	...	...	...	-07
59..	T	08	06	01	-12	33	00	04	03	00	-03	...	...	...	...
60..	T	08	39	-01	01	-04	11	37	-07	-14	14	...	...	...	...
	I	08	25	00	...	...	...	42	03	...	...	-06	20	...	...
61..	T	-03	05	09	-19	-10	-22	-06	03	-02	-20	...	...	...	...

TABLE 6

COSINES BETWEEN THE REFERENCE VECTORS
FOR ALL BATTERIES

(Combination of Matrices $\Lambda'_T \Lambda_T$, $\Lambda'_I \Lambda_I$,
 $\Lambda'_{II} \Lambda_{II}$, and $\Lambda'_{III} \Lambda_{III}$)

	Bat- tery	N	W	S	P	V	M	I	X ₁	X ₂	X ₃
N..	T	1.00									
	I	1.00									
	II	1.00									
	III	1.00									
W..	T	-.19	1.00								
	I	-.26	1.00								
	II	-.26	1.00								
	III	-.10	1.00								
S..	T	-.02	-.06	1.00							
	I	.06	-.09	1.00							
	II	-.07	.00	1.00							
	III	.17	.01	1.00							
P..	T	.09	-.21	-.07	1.00						
	I	-.22	-.29	-.14	1.00						
V..	T	-.20	-.32	-.21	.15	1.00					
	II	-.22	-.27	-.10		1.00					
	III	-.07	-.15	-.28		1.00					
M..	T	.02	-.04	.03	-.19	-.03	1.00				
I..	T	-.05	-.03	-.31	-.07	-.29	-.06	1.00			
	II	.17	-.10	-.25		-.41		1.00			
	III	-.29	-.03	-.56		.02		1.00			
X ₁	T	-.05	-.10	-.11	-.37	.16	-.09	-.11	1.00		
	I	-.43	-.05	-.12	-.05				1.00		
X ₂	T	.11	.00	-.04	-.12	.08	.26	-.10	-.13	1.00	
	II	-.07	-.15	-.29		.23		-.27		1.00	
X ₃	T	-.17	-.11	.30	.12	.12	.12	-.24	-.16	.18	1.00
	III	.01	-.17	.25		-.14		-.46			1.00

TABLE 7

COSINES BETWEEN THE PRIMARY VECTORS
FOR ALL BATTERIES

(Combination of Matrices R_{pT} , R_{pI} , R_{pII} , and R_{pIII})

	Bat- tery	N	W	S	P	V	M	I	X_1	X_2	X_3
N..	T	1.00									
	I	1.00									
	II	1.00									
	III	1.00									
W..	T	.33	1.00								
	I	.46	1.00								
	II	.36	1.00								
	III	.15	1.00								
S..	T	.12	.27	1.00							
	I	.11	.18	1.00							
	II	.17	.23	1.00							
	III	.06	.07	1.00							
P..	T	-.06	.24	.21	1.00						
	I	.43	.45	.22	1.00						
V..	T	.35	.42	.38	-.06	1.00					
	II	.31	.43	.29		1.00					
	III	.10	.22	.32		1.00					
M..	T	.03	.13	.08	.24	.08	1.00				
I..	T	.24	.32	.43	.17	.42	.10	1.00			
	II	.09	.32	.47		.48		1.00			
	III	.37	.16	.54		.21		1.00			
X_1 .	T	.06	.23	.19	.48	-.06	.16	.24	1.00		
	I	.54	.31	.19	.30				1.00		
X_2 .	T	-.17	-.02	.12	.20	-.10	-.18	.07	.18	1.00	
	II	.13	.23	.46		.08		.40		1.00	
X_3 .	T	.23	.10	-.23	-.11	-.03	-.08	.14	.10	-.18	1.00
	III	.28	.22	.13		.17		.57			1.00

Identification.--A line graph was drawn connecting the points representing the loadings of each factor in the tests of Battery I and of each factor in the same tests in the Total Battery. The graph for each factor of Battery I was compared with each of the graphs of the factors in the Total Battery. Each factor in Battery I was identified as that factor in the Total Battery to which it showed the greatest similarity. Figures 1 to 4 have been drawn to illustrate the method of identification used and to indicate the degree of similarity found. The unbroken lines are the graphs for the factors in the Total Battery. The broken lines are the graphs for the factors in the small battery specified. Each corresponding pair of factors is drawn in the same figure; whereas, in the original identification, of course, they were drawn separately.

The similarity of the graphs of the Word Fluency Factor in Figure 1 is typical of the similarities found between factors of both Batteries I and II and their corresponding factors in the Total Battery. In Figure 2 is shown the Perceptual Factor, which, of the identified factors in Battery I, had the least similarity to the corresponding factor in the Total Battery. Figure 3 is the graph of the Space Factor in Battery II and in the Total Battery. It shows the deviations on Tests 35 and 36, which are explained later on in the text. Figure 4 shows the X_3 Factor. This factor showed the least similarity, of any identified factor in the small batteries, to the corresponding factor in the Total Battery.

Of the original ten factors determined by Thurstone (Number, Word Fluency, Space, Verbal, Memory, Induction, Perceptual, X_1 , X_2 , X_3 , the latter three not interpreted) it was possible to identify five factors in Battery I, six in Battery II, and six in Battery III as follows:

Battery I: N, W, S, P, X_1

Battery II: N, W, S, V, I, X_2

Battery III: N, W, S, V, I, X_3

In Battery II the Induction Factor was identified although its loadings were small and in Battery III, three factors with relatively small loadings were identified, Number, Word Fluency, and Induction. None of the unidentified factors had high loadings.

In Battery III none of the factors showed correspondence with the Perceptual Factor of the Total Battery, although several of the tests in this battery had loadings above .40 with that

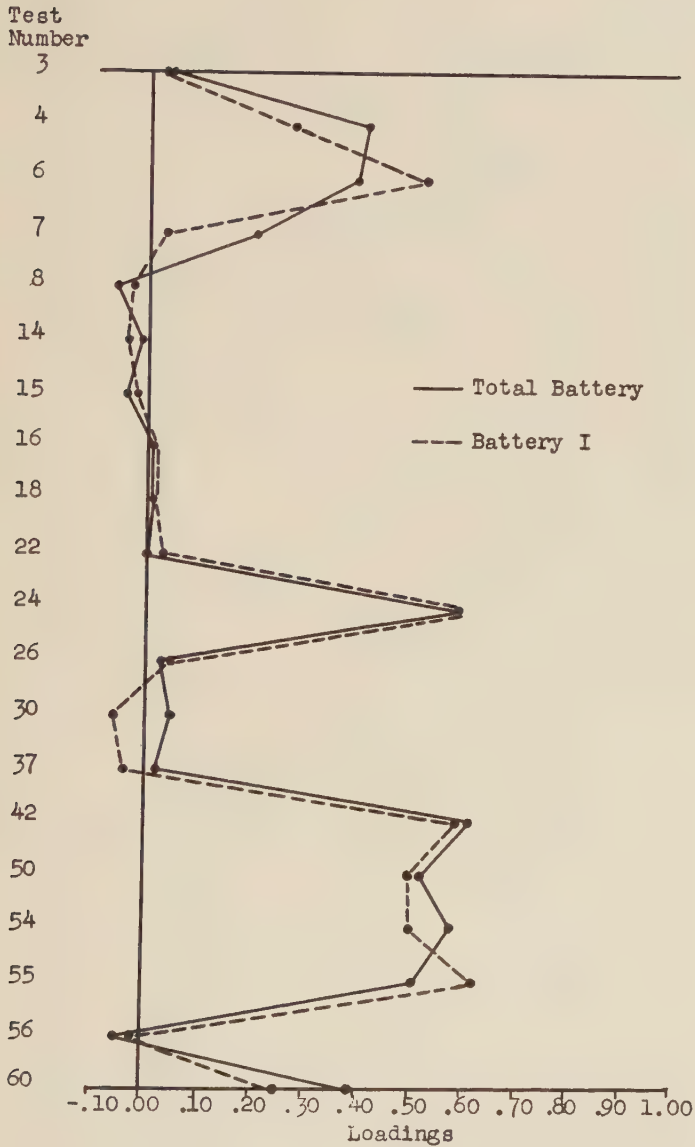


Fig. 1.--Word fluency factor in Total Battery and in Battery I.

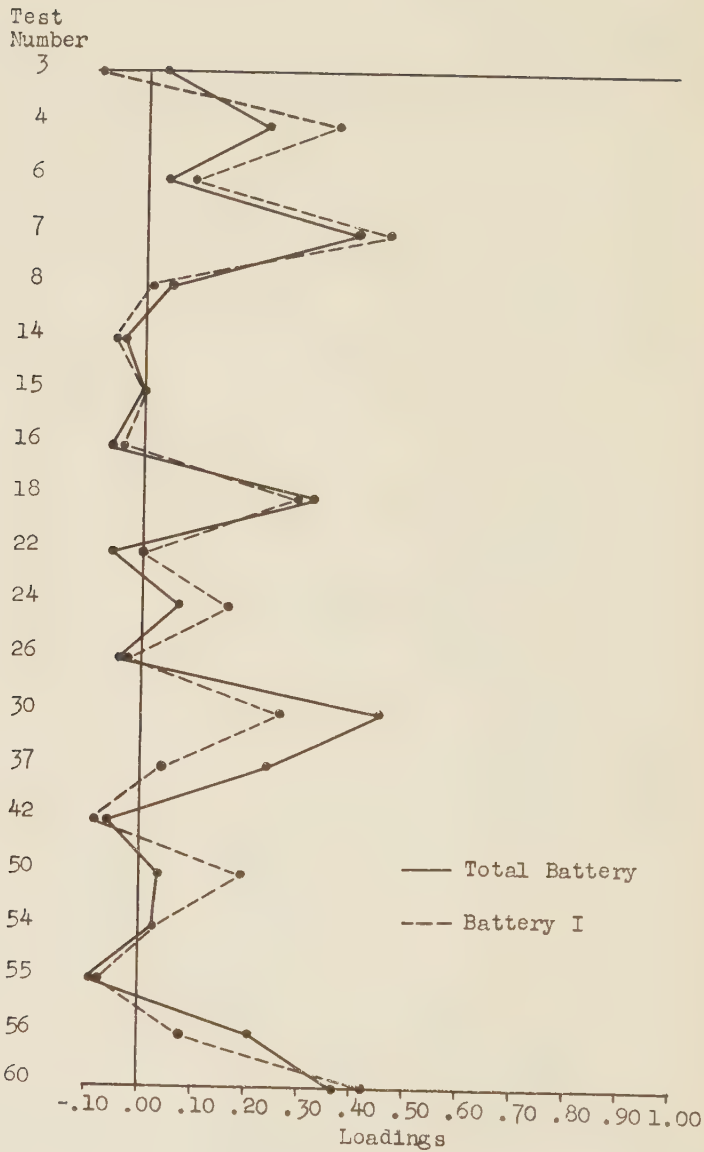


Fig. 2.--Perceptual factor in Total Battery and in Battery I.

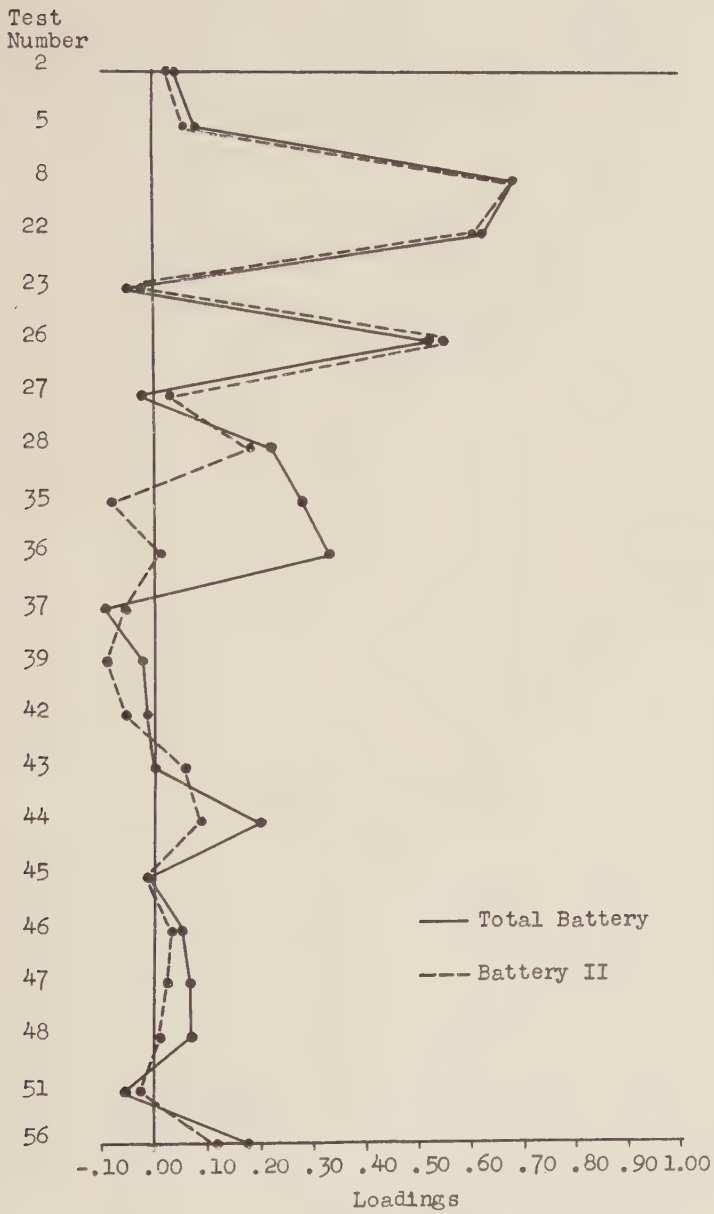


Fig. 3.--Space factor in Total Battery and in Battery II.

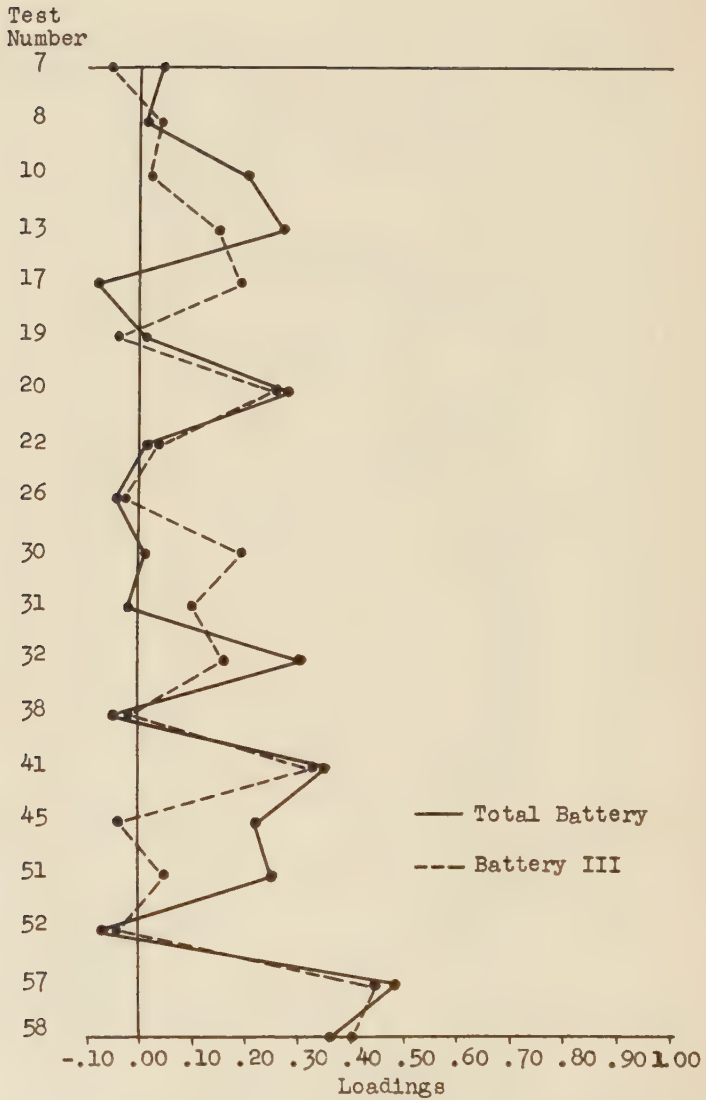


Fig. 4.-- X_3 factor in Total Battery and in Battery III.

factor in the Rotated Factorial Matrix of the Total Battery. The reason for the failure to arrive at the Perceptual Factor in Battery III was that the configuration of points resulting from the selection of this particular battery was such as not to permit the Perceptual Factor to be differentiated from some of the other factors. The battery did not satisfy Thurstone's criterion of Simple Structure.

Let us examine the structure of Battery III in detail. Inspection of Table 1-III shows that the Perceptual Factor column does not have a singular zero relative to each of the other columns. P is defined relative to S and V, but not relative to N, I, W, and X_3 , which were the other factors identified. There are three tests for which the entries are high in column S but zero or very small in column P. Similarly there are three tests with high entries in column V and low entries in column P. But in the pairs N-P, W-P, I-P, and X_3 -P, there are no such entries. It might be expected that Tests 45 and 51, although their X_3 loadings are small (.22 and .25) would help define the Perceptual Plane. This would be true if the X_3 Plane were perfectly defined. But that is not the case. The X_3 Plane is not adequately defined relative to S; so that the small original loadings of X_3 in Tests 45 and 51 are eliminated and hence can not help to define the P Plane.

The lack of Simple Structure in this battery may be explained as follows. The Perceptual Factor was the one which was given the additional emphasis in this battery. Although there were only three tests in which this factor had loadings above .40, there were seven others in which it had loadings ranging from .23 to .36. In order to keep the batteries reasonably small and not to have too much overlapping with the other small batteries the intention was to omit factors N, W, M, I, X_1 , and X_2 , and no tests in which the loadings of these factors were .40 or more were included. However, it happened that the planes of W, M, and I were determined but they had no high loading in any test. This tended to make the other planes indeterminate relative to these three, and it happened that the Perceptual Plane was most affected.

To avoid this situation it would probably have been more reasonable to locate the planes with high loadings orthogonal to all planes with low loadings, instead of adhering strictly to structure.

Relative to X_3 it is also true that no singular zero is

present, but in this case the loadings are not so low. The P Plane would probably have been defined relative to X_3 by Tests 45 and 51 if the X_3 Plane had been adequately defined relative to V by some test such as 43--Proverbs, in which the loading of V is .40 and the loading of X_3 is vanishing. (See Table 1-II.)

The number of clear, or adequately clear defining conditions or singular zeros in Battery III for each factor relative to each other factor are shown in Table 8. Column P shows, as indicated by dashes, that the Perceptual Factor was not defined relative to N, W, and I. Relative to X_3 it had two (starred entry) singular zeros, but these were dependent on X_3 being defined relative to V. Since X_3 was not defined relative to V as indicated by the dash in column X_3 (double-starred entry) the two defining conditions for P relative to X_3 were eliminated.

TABLE 8
NUMBER OF SINGULAR ZEROS IN BATTERY III FOR
EACH FACTOR IN UPPER ROW RELATIVE TO
EACH FACTOR IN LEFT-HAND COLUMN

	N	W	S	V	I	P	X_3
N.....	...	2	2	2	2	...	2
W.....	2	...	2	2	2	...	1
S.....	3	3	...	3	3	3	3
V.....	6	6	6	...	6	6	...**
I.....	1	1	1	1	...	...	1
P.....	9	8	10	9	9	...	6
X_3	9	8	9	3	9	2*	...

In order to show how the Perceptual Plane was destroyed, the diagrams for the four mentioned pairs of factors were plotted and the points representing tests in Battery III were designated by circles. Each of these diagrams showed clearly that the position of the P Plane was considerably changed in going from the Total Battery to Battery III. Figure 5 is the diagram for the Number Factor and the Perceptual Factor. The broken line indicates the position toward which the P Plane was rotated in Battery III. The other diagrams showed similar differences between the location of the Perceptual Plane in the Total Battery and its location in Battery III.

Comparison of loadings.--Comparison of the loadings in

the small batteries with the corresponding ones in the Total Battery in Table 5 shows that the magnitude of the differences is in general quite small. This is particularly true for the large loadings.

In Battery I the three largest discrepancies are .22, .22, and .20. The whole set of discrepancies for this battery may be considered to be not extreme but about what might be expected in view of the fact that in an experimental battery perfect Simple Structure is not obtained, that is, the zero entries are not actual zeros but small positive or negative loadings. In going from one battery to another the planes will of course fluctuate within a small range and thus produce small fluctuations in the loadings.

In Battery II, two exceptions must be noted. Whereas, most of the differences for this Battery are remarkably small, there are two of considerable size. Test 35--Mazes I--and Test 36--Mazes II--each of which had a loading of about .30 with Space in the Total Battery, were found to have insignificant loadings with Space in Battery II. These discrepancies are explained by the fact that the Space Plane was not adequately defined relative to X_2 in Battery II. The change effected in the Space Plane in moving from the Total Battery to Battery II is illustrated in Figure 6. This figure is the scatter diagram of the loadings of S and X_2 in the Total Battery, with the points for Battery II encircled. The broken line indicates the position toward which the S Plane was rotated in Battery II.

In Battery III, out of 114 differences there are seven of absolute value of .20 or more, the largest being .29. Three of these largest differences are with the factor X_3 , which, as has been stated, was not well defined relative to V. All of these seven differences occurred in cases in which the original loading was insignificant, and in only one case did the Battery III loading approach significance. This was in the case of the Test 31, which had a loading of .36 on Induction in Battery III and of .12 on this factor in the Total Battery.

In going from one small battery to another, corresponding loadings also show differences which are not extreme. The largest differences are no larger than those observed in going from a small battery to the Total Battery.

In order to show graphically the extent of agreement between loadings in the large battery and corresponding loadings in

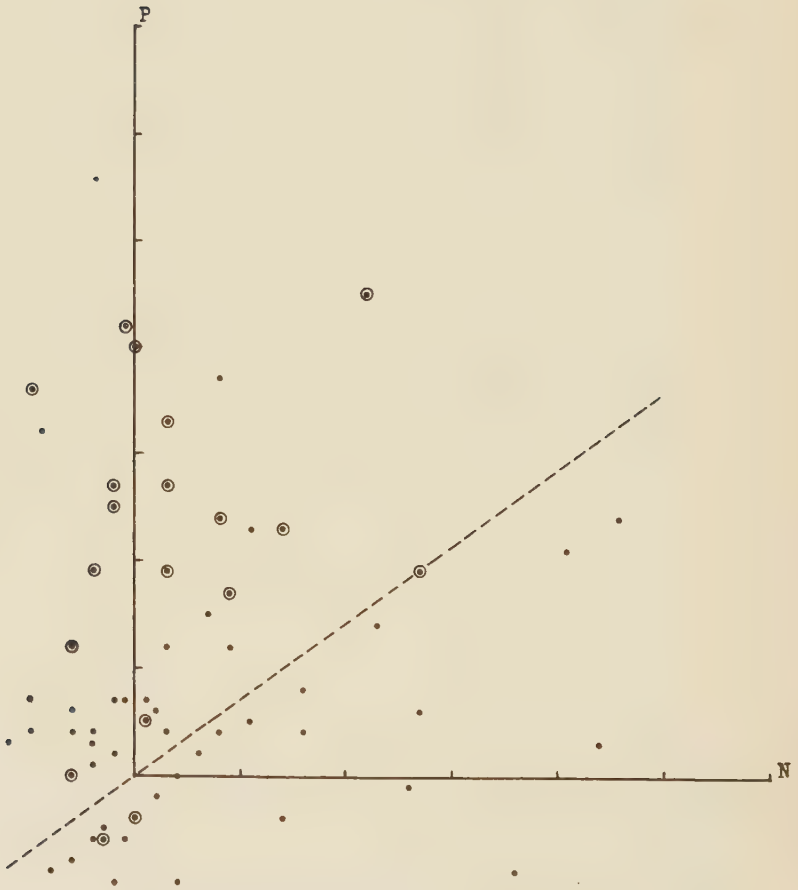


Fig. 5.--Scatter diagram of loadings of number factor and perceptual factor in tests of Total Battery. Encircled points represent tests which were selected for Battery III.

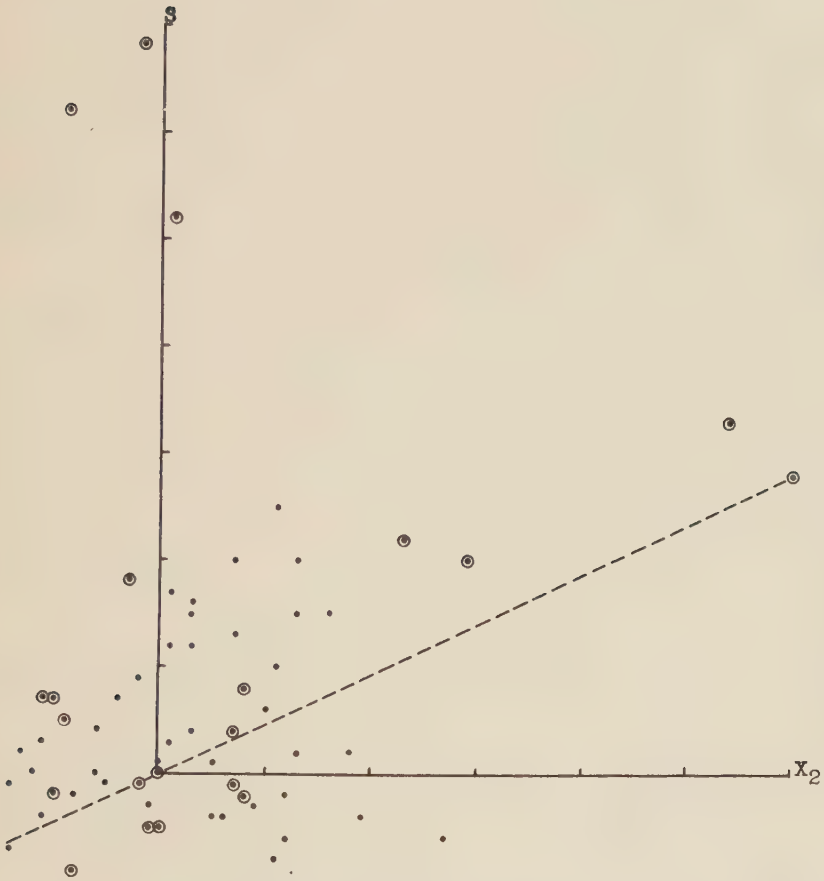


Fig. 6.--Scatter diagram of loadings of X_2 factor and space factor of Total Battery. Encircled points represent tests which were selected for Battery II.

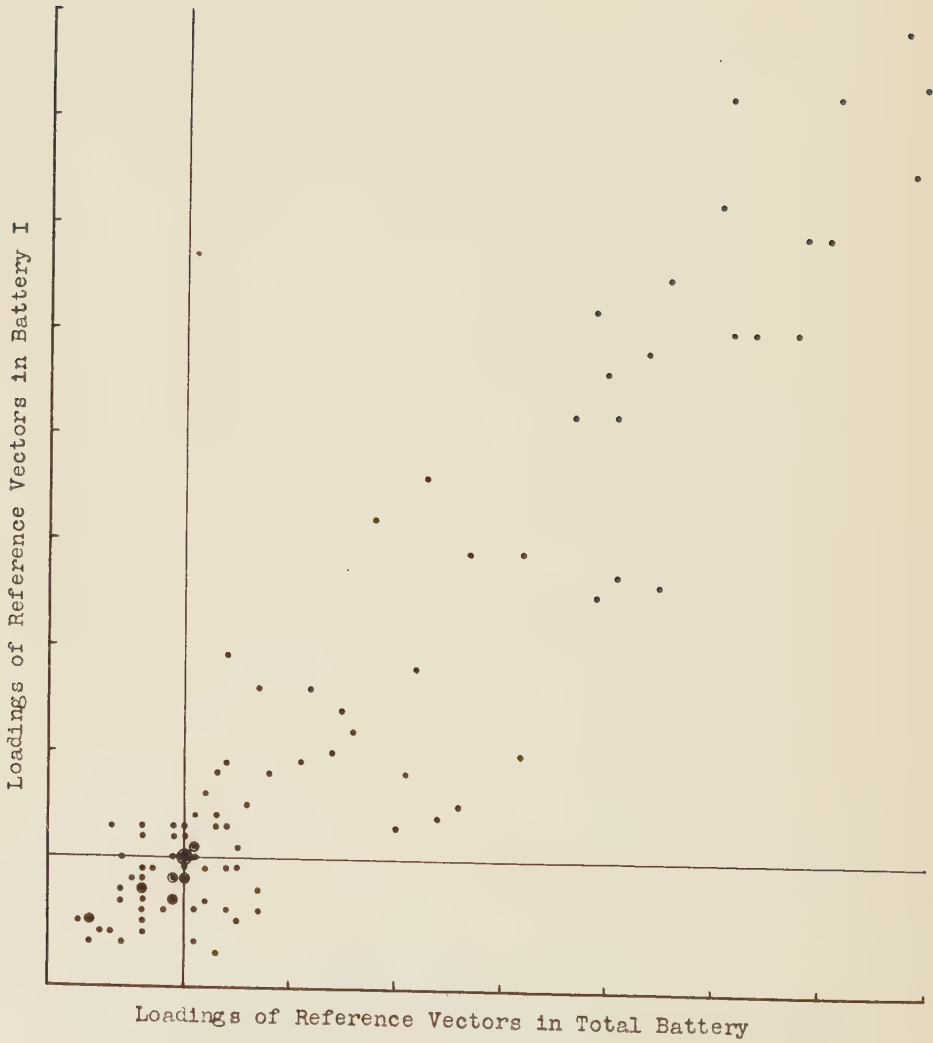


Fig. 7.--Diagram of the two sets of loadings for Battery I

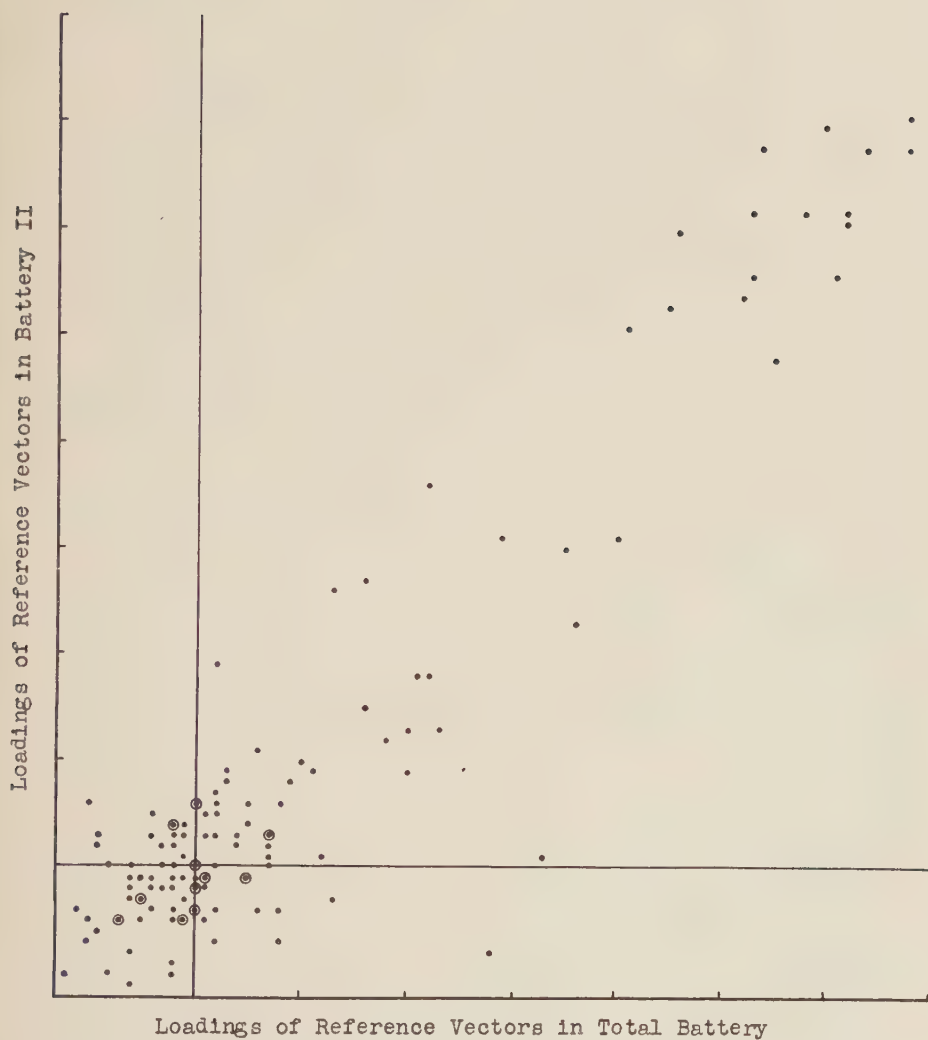


Fig. 8.--Diagram of the two sets of loadings for Battery II

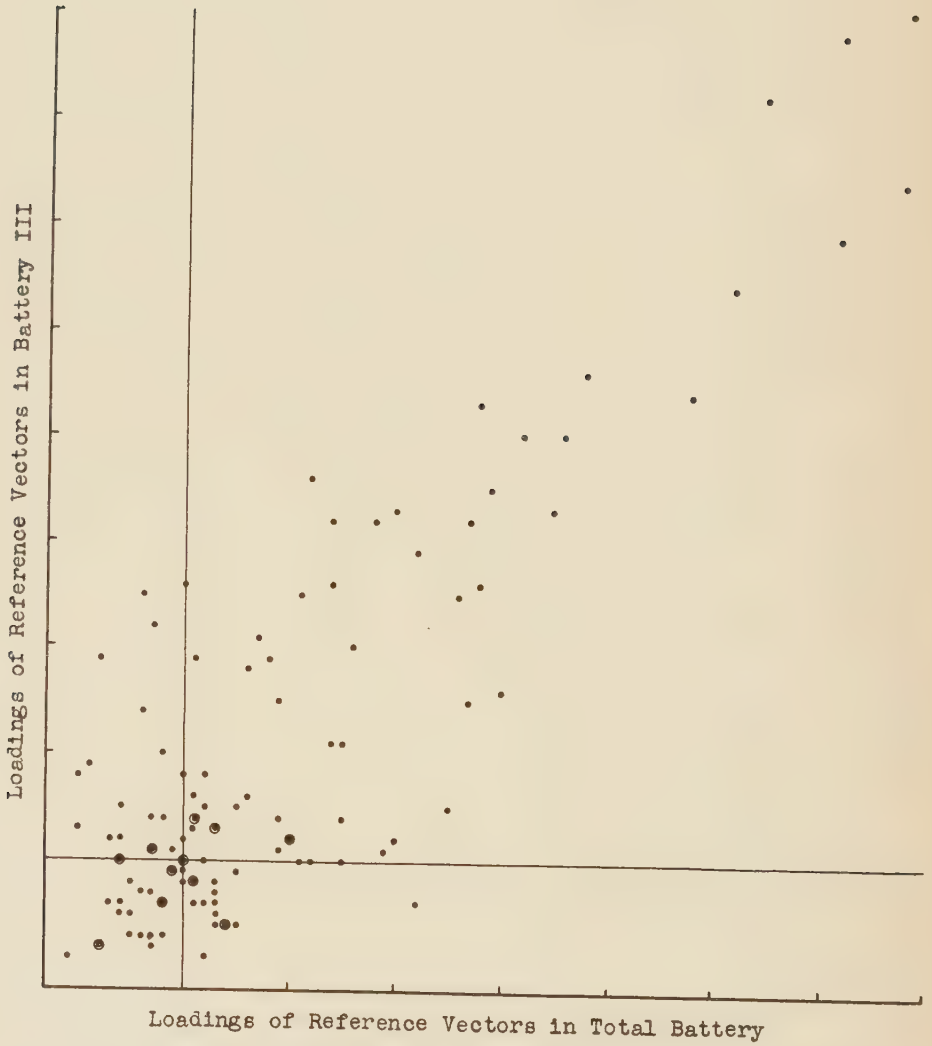


Fig. 9.--Diagram of the two sets of loadings for Battery III

Battery I, a scatter diagram was constructed for them (Fig. 7). Similarly Figure 8 is the scatter diagram for the two sets of loadings of Battery II, and Figure 9 is the scatter diagram for Battery III.

VI. SUMMARY AND CONCLUSIONS

The results of this study show that when sample batteries are selected from a larger battery of tests by the method used in this study, the factors present, derived by Thurstone's Method, in each sample battery can be identified. This is true provided the configuration for each battery has an adequate structure to determine the planes, or, more specifically, provided the configuration satisfies the conditions of Simple Structure.

It follows that Thurstone's Method preserves the invariance of factorial composition of a test; that is, the factorial composition of a test, determined by his method, remains invariant when the test is moved from one battery to another which contains the same common factors. In short, Thurstone's Factorial Method satisfies the criterion which he has set up.

There is no guarantee, of course, in selecting a sample battery from a larger battery which satisfies the conditions of Simple Structure, that these conditions will also be satisfied for the sample battery. In this study two of the three sample batteries did satisfy those conditions rather well, but in the third one the structure was incomplete. One of the planes (Perceptual) with loadings of .40 or more was not discovered because its structure was not adequately defined.

The fact that invariance has been shown to hold for small batteries makes it reasonable to predict that it will also hold for large batteries. In fact, it is more likely to exist in large batteries provided the tests are limited to a given region of abilities, that is, if no new functional unities are introduced.

The demonstration of invariance of factorial composition of a test means that the factors are unique and not dependent on the particular group of tests in a battery. The factors transcend the particular battery in which they are found and may be regarded as functional unities.

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